

Minnesota State High School  
Mathematics League  
Individual Event

3C 1991-92

**Instructions:** The first question should require very little time or computation. It is worth 1 point. Each of the other questions is worth 2 points. Put your answer to each question on the indicated blank on this page.

- \_\_\_\_\_ 1. The long and short edges of a symmetric kite are 3 and  $\sqrt{3}$ , and the short brace BD has length 2 (Figure 1). What is the length of the long brace AC?
- \_\_\_\_\_ 2. For the kite of problem 1, find  $\sin(\angle BAD)$ .
- \_\_\_\_\_ 3. Find a number which, when squared, gives  $-2 + 2i\sqrt{3}$ ?
- \_\_\_\_\_ 4. [ARML, 1991] The area of  $\triangle ABC = 6$  and its hypotenuse AC has length 8 (Figure 4). Find  $\sin 2A$ .

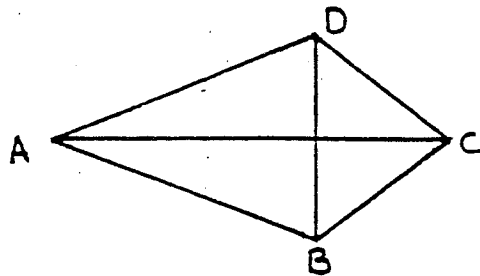


Figure 1

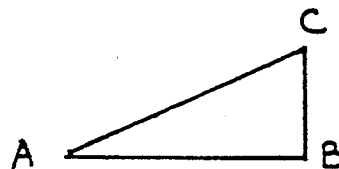
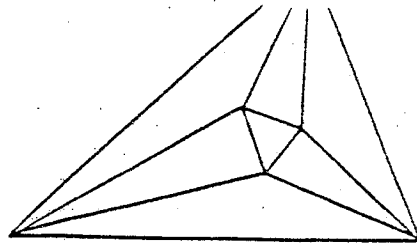


Figure 4

Name \_\_\_\_\_

School \_\_\_\_\_



Minnesota State High School  
Mathematics League  
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3C 1992-93

**Instructions:** The first question should require very little time or computation. It is worth 1 point. Each of the other questions is worth 2 points. Put your answer to each question on the indicated blank on this page.

\_\_\_\_\_ 1. Give in rationalized form the  $\cos (\operatorname{Arctan} (-2))$ .

\_\_\_\_\_ 2. Express  $\operatorname{Arcsin} \left( -\frac{4}{5} \right)$  as the sum of a multiple of the angle  $\beta$  pictured in Figure 2 and a multiple of  $\pi$ .

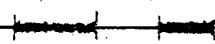
$x_1 =$

$x_2 =$

$x_3 =$

$x_4 =$

3. The graph of  $y = \cos 2x - \sin 2x + 1$  is drawn in Figure 3, with  $x_1$ ,  $x_2$ ,  $x_3$ , and  $x_4$  marking the points where the graph crosses the  $x$  axis between 0 and  $2\pi$ . Find  $x_1$ ,  $x_2$ ,  $x_3$  and  $x_4$  in radian measure.

4. The  $x$  axis in Figure 4 is marked in intervals of  $\frac{\pi}{12}$ ,  $0 \leq x \leq \pi$ . Blacken the interval or intervals (  ) in which you think there is a value of  $x$  such that  $\sin x + \cos x = \tan x$ . If you blacken two intervals, even adjacent intervals, that will be taken to mean that you believe there is a solution to the equation in each interval.

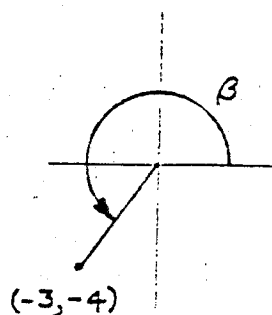


Figure 2

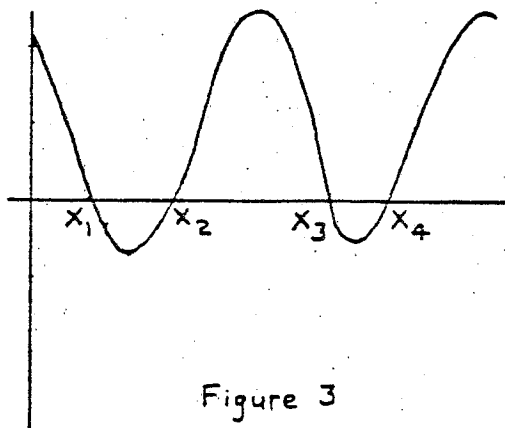


Figure 3

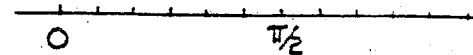
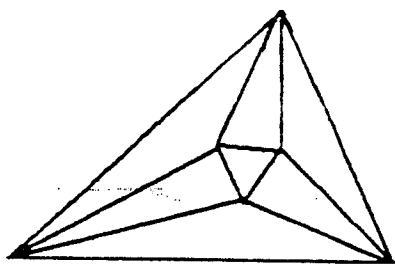


Figure 4

Name \_\_\_\_\_

School \_\_\_\_\_

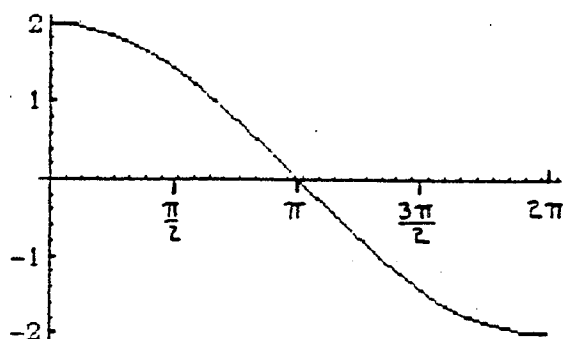


# Minnesota State High School Mathematics League

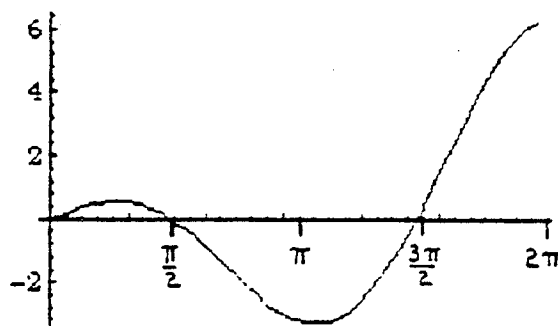
## Individual Event

### 1993-94 Event 3C

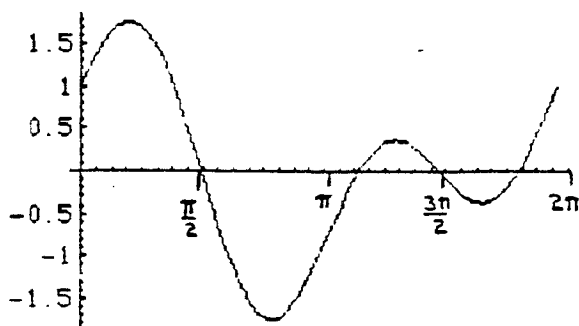
The first question is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. In each case you are given a graph and a list of equations, one of which corresponds to the graph. Circle what you believe to be the corresponding equation. You have 12 minutes for this event.



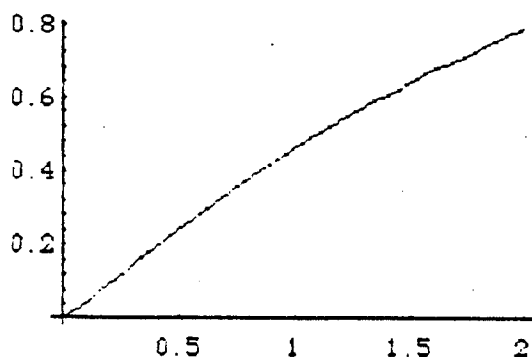
- 1.
- (a)  $y = \cos \frac{x}{2}$
  - (b)  $y = \cos 2x$
  - (c)  $y = \frac{1}{2} \cos 2x$
  - (d)  $y = 2 \cos \frac{x}{2}$
  - (e)  $y = 2 \cos x$



- 2.
- (a)  $y = x + \sin x$
  - (b)  $y = x + \cos x$
  - (c)  $y = x \sin x$
  - (d)  $y = x \cos x$



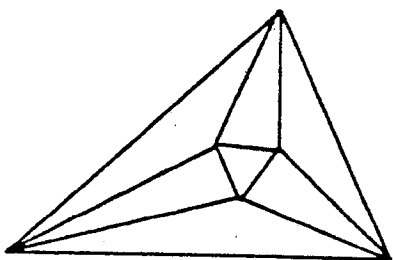
- 3.
- (a)  $y = \cos x + \sin x$
  - (b)  $y = \cos x + \sin 2x$
  - (c)  $y = \cos x + 2 \sin x$
  - (d)  $y = 2 \cos x + \sin x$



- 4.
- (a)  $y = \text{Arcsin} \frac{x}{2}$
  - (b)  $y = \frac{1}{2} \text{Arcsin} x$
  - (c)  $y = \text{Arc tan} \frac{x}{2}$
  - (d)  $y = \frac{1}{2} \text{Arc tan} \frac{x}{2}$

Name \_\_\_\_\_

Team \_\_\_\_\_

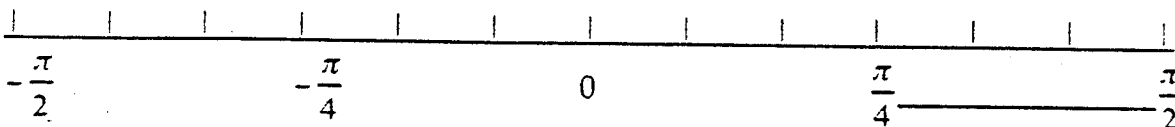


# Minnesota State High School Mathematics League Individual Event

## 1994-95 Event 3C

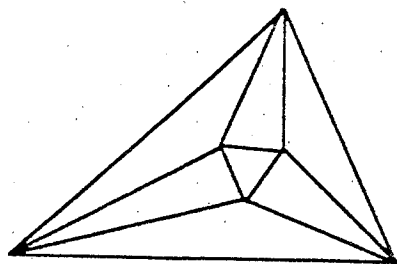
The first question is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

- \_\_\_\_\_ 1. If  $\alpha = \text{Arcsin}\left(-\frac{4}{5}\right)$  and  $\beta = \text{Arccos}\left(\frac{4}{5}\right)$ , what is  $\sin(\alpha - \beta)$ ?
  
- \_\_\_\_\_ 2. If  $r$  is a real number,  $0 < r < 1$ , what is  $\sin(\text{Arccos } r + \text{Arccos } \sqrt{1 - r^2})$ ?
  
- \_\_\_\_\_ 3. Express as a rational multiple of  $\pi$  all  $x$ ,  $0 \leq x \leq 2\pi$ , that satisfy  $2\sin^2 x + 3\cos x = 0$ .
  
- \_\_\_\_\_ 4. The  $x$ -axis below is marked in multiples of  $\frac{\pi}{12}$ . Blacken those intervals in which you think there is a value of  $x \neq 0$  such that  $\sin 2x = \text{Arcsin } x$ . If you blacken two intervals, even adjacent intervals, that will be taken to mean that you believe there is a solution in each interval.



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Name

\_\_\_\_\_  
Team



# Minnesota State High School Mathematics League Individual Event

## 1995-96 Event 3C

The first question is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

1. Give, rounded correct to four places after the decimal,  $\sin \alpha$ , given that  $\sin 2\alpha = \frac{4}{5}$   
\_\_\_\_\_ and that  $0 < \alpha < 45^\circ$ .

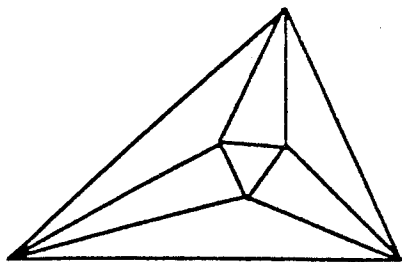
2. If  $0 < a < 1$ , find  $\sin\left(\operatorname{Arctan} a + \operatorname{Arctan} \frac{1-a}{1+a}\right)$ , expressed as a decimal rounded correct  
\_\_\_\_\_ to four places after the decimal.

3. Simplify  $\tan 15^\circ \tan 30^\circ \tan 45^\circ \tan 60^\circ \tan 75^\circ$   
\_\_\_\_\_

4. The base of an isosceles triangle has length 2 and the cosine of the apex angle is  $1 - a^2$  where  $0 < a < 1$ . Find the length of a leg of the triangle in terms of  $a$ .  
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Name

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Team



# Minnesota State High School Mathematics League

## Individual Event

### 1996-97 Event 3C

The first question is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

1.  $\triangle ABC$  has side lengths  $a$ ,  $b$ , and  $c$ , with  $a$  opposite  $\angle A$ , etc. You are given the measures of  $\angle A$ ,  $\angle B$ , and side length  $c$ . Give in the form  $b = c$  (the quotient of two sine functions) a formula by which  $b$  may be determined from what is known.

\_\_\_\_\_

2. The sides of a triangle have lengths 4, 5, and 6. Find, to the nearest tenth of a degree, the measure of the largest angle.

\_\_\_\_\_

3. Given that  $\tan \alpha = a$  and  $\tan \beta = \frac{a}{2}$ , express  $\tan(2\alpha + \beta)$  as a rational function of  $a$ .

\_\_\_\_\_

4. E marks the midpoint of side BC of a square piece of paper ABCD (Figure 4a). The paper is then folded so that D coincides with E, making AD cross the top edge at T (Figure 4b). Find, to the nearest tenth of a degree, the measure of  $\angle BTE$ .

\_\_\_\_\_

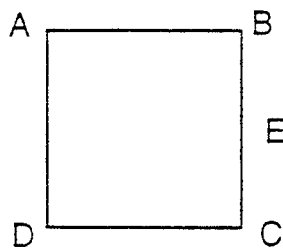


Figure 4a

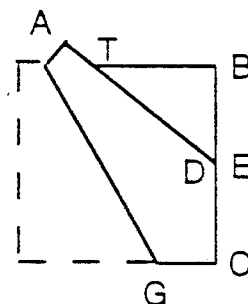
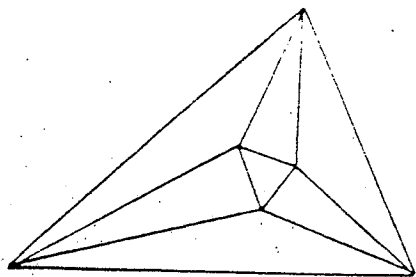


Figure 4b

Name \_\_\_\_\_ Team \_\_\_\_\_



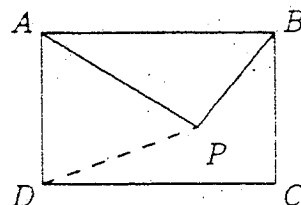
Minnesota State High School  
Mathematics League  
*Individual Event*

**1997-98 Event 3C**

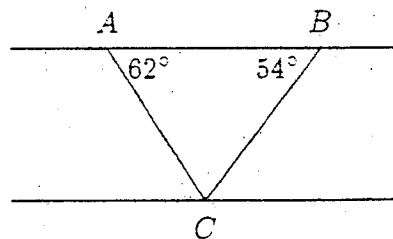
The first question is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

- \_\_\_\_\_ 1. Find all values of  $\theta$  (in degrees),  $0 < \theta < 90^\circ$ , such that  $\cos \theta - \sqrt{3} \sin \theta = 0$ .
- \_\_\_\_\_ 2. If  $\cos \theta = \tan \theta$ , find the value of  $\sin \theta$ , rounded to four places after the decimal point.

- \_\_\_\_\_ 3. In the rectangle  $ABCD$ ,  $AD = 10$  and  $CD = 15$ .  $P$  is a point inside the rectangle such that  $PB = 9$  and  $PA = 12$ . Calculate the length of  $PD$ .

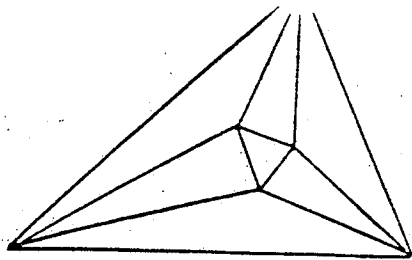


- \_\_\_\_\_ 4.  $A$  and  $B$  are on one side of a straight river, 300 feet apart. Point  $C$  is on the opposite side and is situated so that  $\angle CAB = 62^\circ$  and  $\angle ABC = 54^\circ$ . Find the width of the river to the nearest tenth of a foot.



Name \_\_\_\_\_

School \_\_\_\_\_

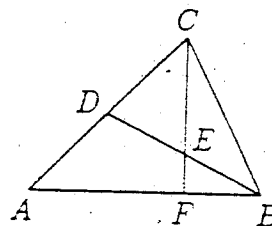


Minnesota State High School  
Mathematics League  
*Individual Event*

**1998-99 Event 3C**

The first question is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

- \_\_\_\_\_ 1. Give, rounded correct to three places after the decimal, the measure of angle  $\theta$ ,  $90^\circ < \theta < 180^\circ$ , so that  $5 \cos 2\theta = -3$ .
- \_\_\_\_\_ 2. Find the largest value of  $\theta$ ,  $0 < \theta < 360^\circ$ , such that  $\sin \theta = \frac{\sec \theta}{4}$ .
- \_\_\_\_\_ 3. A regular 12-sided polygon whose sides are each 10 units long is inscribed in a circle. What is the radius of this circle? Give your answer rounded correct to four places after the decimal.
- \_\_\_\_\_ 4. In the figure,  $\angle CAB = 37^\circ$ ,  $\angle DBA = 25^\circ$ ,  $CF$  is perpendicular to  $AB$ ,  $ED = 10$  units, and  $E$  is the intersection of  $CF$  and  $BD$ . Find the length of  $CE$ , rounded correct to four places after the decimal.



Name \_\_\_\_\_  
School \_\_\_\_\_